GENERALIZATION OF EXPERIMENTAL DATA ON HEAT AND MASS TRANSFER DURING THE EVAPORATION AND SUBLIMATION OF DIFFERENT SUBSTANCES IN A FORCED STREAM

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Experimental data on heat and mass transfer during evaporation (sublimation) are generalized on the basis of the usual methods of similarity theory. The results of an analysis of the author's experimental data are presented.

One of the main difficulties arising during the application of similarity theory to a specific complicated process, such as combined heat and mass transport, for example, consists in the construction of a sufficiently complete and physically correct system of criteria.

As an example of a physical system let us examine the quasistationary turbulent flow of a binary mixture at a "semipermeable" surface of evaporation (sublimation). We will assume that the sources of the physicochemical transformation are uniformly distributed over the surface and that the transformation itself takes place in thermodynamic equilibrium. We will disregard the secondary molecular processes (Dufour and Soret effects). Radiation is not important. The case of evaporation into a vacuum is not touched upon.

We will assume that the mixture and its components are close in properties to an ideal gas, $i = c_pT$. The heat capacity and gas constant of the mixture depend on the relative concentrations of the components, and the density of the mixture depends on the temperature in addition. Although there is no doubt that the transport coefficients also depend on the content of the components, in many cases this dependence is not very marked. The differential equations for the boundary layer are well known [1] for the conditions formulated.

In order for the problem in principle to admit of a unique solution the values of all the variables must be given at the boundaries of the system (or some equivalent conditions). In particular, we assume that the temperature T_w and the partial pressure p_{1w} at the surface of evaporation are known.

By reducing the existing relationships to dimensionless form one can use the method of scaling transformations. Here the scales of the values are their values given by the conditions of uniqueness. If there are at least two such values (such as the stream and the wall, for example), none of which is equal to zero or can be reduced to zero by a shift in the frame of reference, then a determining simplex, representing the ratio of these parametric independent values or scales, appears in the criterial relationship.

Thus, for the conditions formulated above we obtain the determining dimensionless complexes Re, M, Pr, and Sc and the determining simplexes $(p_{1w} - p_{1f})/p$, p_{2f}/p, $(T_f - T_w)/T_f$, R_1/R_2, and c_{p1}/c_{p2}.

If the boundary conditions at the wall are written as

$$q = jr, \tag{1}$$

(for "adiabatic" evaporation), where

$$q \equiv \alpha (T_f - T_w) = \left[\lambda \left(\frac{\partial T}{\partial n}\right)\right]_w, \quad j \equiv \beta (p_{1w} - p_{1f}) = \left[-K_{12}\frac{\rho^2}{\rho_2}\left(\frac{\partial \bar{\rho}_1}{\partial n}\right)\right]_w, \qquad (2)\,(3)$$

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then one can obtain from (1) the criterion of phase transformation

$$K \equiv \frac{r}{c_p (T_f - T_w)} .$$

It is obvious that this dimensionless complex serves the same purpose as the relationship (1), i.e., it is a measure of the interdependence of the intensities of heat and mass transfer. On the basis of (2) and (3) it can also be asserted that this is a measure of the interdependence of the temperature field and the density fields at the surface of transformation. Therefore the following statement is also valid: with the assignment of the boundary condition (1) the need to assign p_{1W} falls off, and consequently K is included among the determining criteria in place of the simplex $(p_{1W} - p_{1f})/p$. (It would not be out of place to observe that the determining criterion of mass exchange should also be taken here in such a way that knowledge of p_{1W} is not required. The criterion Nu_D does not satisfy this requirement.)

Although in principle it should be asserted that independently of how the determining complex is taken (for mass exchange or for heat exchange) the system of determining criteria for the process of combined heat and mass transfer under consideration must remain the same, the nature of the effect of the determining criteria can undoubtedly be quite different in this case. One can see from the differential equations for the boundary layer – for mass balance of the vapor and for energy – that the interdependence of their solution is manifested primarily through the density of the mixture which depends both on the concentrations and on the temperatures. Several particular cases of criterial relationships can be obtained here depending on the concrete circumstances of the process.

It is known that at low intensities of evaporation (sublimation) the criterial dependence for the diffusional Nusselt number appears analogous (down to the matching of the coefficients of the equation) to the dependence for "pure" heat exchange of incompressible liquids under the same hydrodynamic conditions (in this case the Schmidt numbers emerges as an analog of the Prandtl number). It is clear that in this case, apart from the analogous structure of the differential equations of vapor mass balance and of energy (obtained from the discarding of unimportant terms from the energy equation), the former must be solved independently from the latter (although the question is whether a solution is possible in principle).

The effect of the factor of variability of the density of the mixture increases with an increase in the intensity of evaporation, since the drops in concentrations and temperatures increased simultaneously with it. This effect should be considered as considerably greater than during "pure" heat exchange, possibly because the total density enters into the differential equation of vapor mass balance in the form of a derivative. Here additional determining criteria appear in the critical dependence.

We conducted experiments on the porous evaporation of water, acetone, and ethyl alcohol (Π = 43%) and the sublimation of a number of substances from the surface of the flat transition element of a manifold which constricts an air stream (diaphragm). The method of conducting the experiments is described in [2, 9]. The mode of flow is turbulent and subsonic. The evaporation of the liquids had a nature close to adiabatic, i.e., practically all the heat transferred from the gas went into evaporation. The sublimation had a markedly nonadiabatic nature. As a consequence of this there was no opportunity to treat the entire collection of experimental data using the phase transition criterion since in the form indicated it is a criterion only of adiabatic evaporation. However, it was possible to generalize the experimental data on evaporation and sublimation using the determining simplices

$$\frac{p_{1w} - p_{1f}}{p}, \quad \frac{T_f - T_w}{T_f} .$$

The simplex of pressures (Fig. 1a) plays the main role in this. The dependence represented in Fig. 1a can be described by the equation

$$\Pi_D = 0.126 \, \mathrm{Re}^{0.6} \left(\frac{p_{1w} - p_{1f}}{p} \right)^{2/3} . \tag{4}$$

The scatter of the experimental points here is considerable (±30%), although it can be decreased since the stratification of the experimental points as a function of the temperature simplex is clearly revealed on the graph.

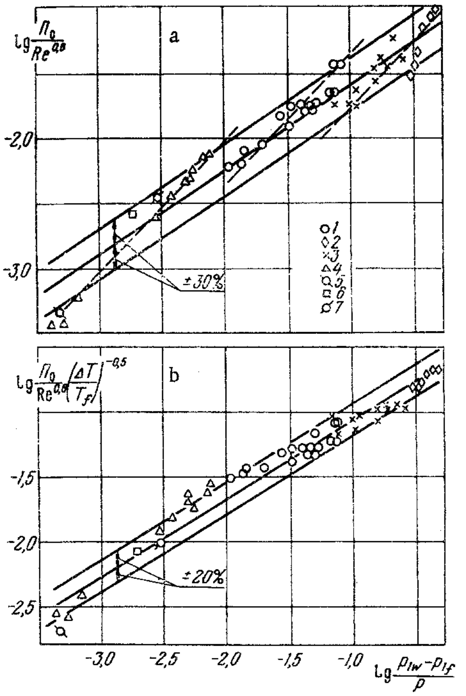

Fig. 1. Dependence of dimensionless intensity of mass transfer on simplices of partial pressures (a) and temperatures (b) for average values over the surface of a flat transition element of a manifold at d / D = 0.2; L / D = 1.48-3.0: 1) water; 2) acetone; 3) alcohol; 4) naphthalene; 5) diphenyl; 6) benzoic acid; 7) salicyclic acid.

The dependence illustrated in Fig. 1b is described by the equation

$$\Pi_D = 0.36\,\mathrm{Re}^{0.6}\left(\frac{p_{1w}-p_{1f}}{p}\right)^{0.6}\left(\frac{T_f-T_w}{T_f}\right)^{0.5}. \tag{5}$$

The scatter of the experimental points for this dependence is ±20%. Given the very wide range of variation in the parameter $(p_{1w}-p_{1f})/p = 4\cdot10^{-4}$–$0.44$ it cannot be considered as very great. It lies within the limits of accuracy of the experimental data. The temperature simplex varied in the range of 0.017–0.2.

Equations (4) and (5) pertain to values of the intensity of evaporation (sublimation) j and the temperature T_w averaged over the surface (averaging with respect to area, see [2, 9] for more detail). The fraction of radiant flux to the surface was not more than 6%. The values entering into the similarity criteria are taken from the state of the mixture at the entrance to the working section. The values T_f and p_{1f} refer to there. The Prandtl number in the experiments was practically constant and equal to 0.7. The criterion p_{2f}/p, which characterizes the vapor content of the substance evaporated (sublimed), at the entrance was either equal to unity or (for water) differed insignificantly from it (p_{1f}/p was not more than 0.016). The parameters Sc, R_1/R_2, and c_{p1}/c_{p2} were also not reflected in this treatment.

It is interesting to note that if the sublimators, water, and the highly volatile liquids are examined separately in Fig. 1a then Π_D is approximately proportional to $(p_{1w}-p_{1f})/p$ to the first power, which is equivalent to the absence of a dependence of Nu_D on this simplex. With allowance for what has been obtained it is clear that this is a result of the joint effect of the simplex $(p_{1w}-p_{1f})/p$ and the temperature simplex.

The very possibility of representing the experimental data on sublimation and on porous evaporation (which was conducted in the absence of recessing of the evaporation surface) by a single dependence indicates the absence of fundamental differences in the courses of these processes. In particular, it can be assumed that the effective surface of evaporation under such conditions coincides with the geometrical surface of the substance, which is evidently connected with two circumstances: the film of liquid at the surface is continuous and the evaporation from the depressions and protrusions of the meniscuses obeys the law of cosines. (Experiments on the porous evaporation of water which we conducted under the same conditions but with material of 30% porosity are described in [7]. The effect of these differences in porosity (43 and 30%) was not noticeable.)

The data on the evaporation of water, acetone, and ethyl alcohol presented in Fig. 1 were analyzed earlier [2] as a function of the criterion of phase transformation

$$\Pi_D = 0.41\,\mathrm{Re}^{0.6}\,K^{-0.75}. \tag{6}$$

A comparison of (4) and (5) with (6) thus actually confirms the equivalence of the criterion K and the simplex $(p_{1w} - p_{1f})/p$ mentioned above. This circumstance also has a direct connection with the ratio α/β, since in place of (1) one can write

$$\alpha (T_f - T_w) = \beta (p_{1w} - p_{1f}) r. \tag{7}$$

Hence

$$\frac{\alpha/c_p}{\beta p} \frac{c_p (T_f - T_w)}{r} = \frac{p_{1w} - p_{1f}}{p}. \tag{8}$$

If

$$\frac{\alpha/c_p}{\beta p} = \text{const}, \tag{9}$$

then the criteria K and $(p_{1w} - p_{1f})/p$ become completely interchangeable.

If it is understood that Eqs. (7) and (8) pertain to the local values of α, β, and T_w while T_f and p_{1f} are taken with respect to the state at the entrance to the evaporation chamber (i.e., they are fixed as unchanged parametric values in a given problem) then the constancy of the local temperatures t_w of the evaporation surface follows from the condition (9) (and vice versa) in the course of the process of adiabatic saturation of the gas (drying agent) by the vapors of a given evaporating liquid. Thus, the condition (9) proves to be related to the condition

$$t_w = t_l = t_M = \text{const}. \tag{10}$$

As shown in [5], by using only the condition (10) and the equation of energy balance of the adiabatic evaporation chamber one can obtain the following relationship in turn:

$$\frac{c_{p2}(T_f - T_w)}{r} = \frac{R_2}{R_1} \frac{p_{1w}}{p_{2w}} - x_f \frac{i_{1f} - c_l t_w}{r}, \tag{11}$$

where x_f is the mass content of vapor in the dry gas at the entrance to the chamber; i_{1f} is the enthalpy of the vapor at the entrance; T_f is the temperature of the mixture at the entrance.

It is interesting that at $x_f = 0$ Eq. (11) coincides with a dependence of the same kind but obtained from an examination of concentration and temperature fields with the condition of their similarity ($Pr = Sc = 1$) and several other assumptions [11].

In conclusion, let us examine one of the possibilities for using the data on "pure" heat exchange for calculating the heat exchange in the presence of a phase transformation with the supply of mass to the boundary layer. It was shown in [2] that at least for the evaporation intensities studied the effect of the hydrodynamics of the main stream on "pure" heat exchange and on heat exchange with evaporation is the same. The effect of the supply of mass to the boundary layer from the source of the phase transformation can be characterized for adiabatic evaporation primarily by the criterion of phase transformation and, it appears from our data, by the complex $Sc\, R_1/R_2$ [2].

By comparing the dependence presented in [2] for heat exchange during the evaporation of different liquids from the surface of the flat transition element of a manifold with the data of [10] on "pure" heat exchange obtained on the same apparatus under completely analogous hydrodynamic conditions we have ($K = 6$-250)

$$\alpha/\alpha_0 = 0.26 K^{0.3} (Sc\, R_1/R_2)^{-1}. \tag{12}$$

We obtain a result very close to this by conducting the same comparison for Vainberg's data on the evaporation of acetone, alcohol, benzene, and water during longitudinal external flow over a plate [6]. By comparing them with the well-known expression for "dry" heat exchange under analogous conditions we obtain

$$\alpha/\alpha_0 = 0.37\ K_l^{0.3}. \tag{13}$$

The author of [6] used the modified phase transformation criterion $K_l \equiv r/c_l(T_f - T_w)$, which varies in the range of 4-19. The inclusion of the heat capacity of the liquid c_l in this criterion is insufficiently justified in our opinion and evidently occurred because of an initial overestimation of the fraction of heat going into the heating of the newly entering liquid. If c_l is converted to the usually used form of the phase

TABLE 1. Values of Complex Sc R_1/R_2 and Heat Capacity c

Liquid studied	Sc R_1/R_2	c_l, kcal /kg · deg	c_{p1}, kcal /kg · deg
Water	1,0	1,0	0,45
Acetone	0,6	0,5	0,35
Alcohol	0,8–0,85	0,6	0,35
Benzene	0,6	0,4	0,25

TABLE 2. Results of Calculations

Liquid evaporated	K	α/α_0 from Eq.		
		(12)	(14)	(15)
Water	30	0,72	0,67	0,58
	200	1,27	1,18	1,24
Acetone	10	0,85	0,60	
	30	1,18	0,82	
Ethyl alcohol	10	0,61	0,56	
	50	0,98	0,91	
	100	1,21	1,12	

transformation criterion with the substitution of $c_p = 0.24$ kcal / kg · deg then in place of (13) we have

$$\alpha/\alpha_0 = 0.24K^{0.3}\,c_l^{-0.3}\,, \tag{14}$$

where c_l is accordingly also in kcal / kg · deg and K is in the range of 9-80.

The values of the complex Sc R_1/R_2 and of the heat capacity c_l are very close for the liquids studied (Table 1).

If one also takes into account the small ranges of variation of these parameters, their closeness to unity, and the error in conducting the experiments, which in combination can also be reflected in the exponents of these values, then Eqs. (12) and (14) must be acknowledged to be identical with respect to those effects which they take into account.

Finally, if we again turn to the critical dependence obtained by the authors of [3] for heat exchange during the evaporation of water from the porous wall of a rectangular channel and compare it with the corresponding dependences for "dry" heat exchange during turbulent flow we will have (K = 28-400)

$$\alpha/\alpha_0 = 0.149K^{0.4}\,. \tag{15}$$

Although the exponent of K differs in this case, which may be connected with the properties of the flow in the initial section of the slotted channel, the numerical results of the calculations for water are rather close for the three cases (Table 2).

As seen from Table 2, the calculations from Eqs. (12) and (14) for the liquids studied give marked differences only for acetone. In this case the Schmidt number was computed from the diffusion coefficient of acetone in air given in [12]: 0.109 cm²/sec. The results of the calculation for acetone agree much better if the value Sc = 1.6 given in [13] is used. Then from Eq. (12) for K equal to 10 and 30 we have α/α_0 equal to 0.65 and 0.90, respectively.

The general tendency reflected by a dependence of the type of (12) is for the heat-exchange coefficient to fall with an increase in the transverse flow of mass (the criterion of phase transformation decreases in this case). At the same time, as seen from Table 2, at rather low intensities of evaporation the heat-exchange coefficients are higher than with "dry" heat exchange. A number of investigators ([8] et al.) in their time were able to detect only the latter case, which served as the source of discussion. Relatively recently Mugalev discovered an analogous increase in the heat-exchange coefficients during light blowing of gases [4] and logically explained them by the discreteness developing in this case in the jets of gas blowing through the porous membrane which leads to disturbances in the structure of the boundary layer. In the processes of evaporation and sublimation the similarity pattern evidently will be primarily connected with the microsource nature of the phase transformation.

NOTATION

$\Pi_D = jD/K_{p1}p = Nu_D[(p_{1w}-p_{1t})/p]$	is the criterion of intensity of mass transfer;
j	is the density of mass flow of vapor;
$K_{12} = K_{p1}R_1T$	is the coefficient of interdiffusion of vapor and air;
p	is the pressure;
R	is the gas constant;
T, t	are the temperature;
$K \equiv r/c_p(T_f-T_w)$	is the criterion of phase transformation;
r	is the heat of phase transition;

c	is the heat capacity;
$Sc \equiv \nu / K_{12}$	is the Schmidt number;
$Re \equiv wD/\nu$	is the Reynolds number;
Π	is the porosity;
i	is the enthalpy;
ρ	is the density;
D	is the diameter of supply manifold;
d	is the diameter of opening in transition element of manifold;
L	is the distance from entrance (uniform field of velocities and temperatures) of working section to transition element;
μ	is the molecular mass;
α	is the heat-exchange coefficient.

Subscripts

w	denotes the wall;
f	denotes the impinging stream;
l	denotes the liquid;
s	denotes the saturation;
m	denotes the "moist" thermometer;
0	denotes the "dry" heat exchange;
1	denotes the vapor;
2	denotes the air.

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